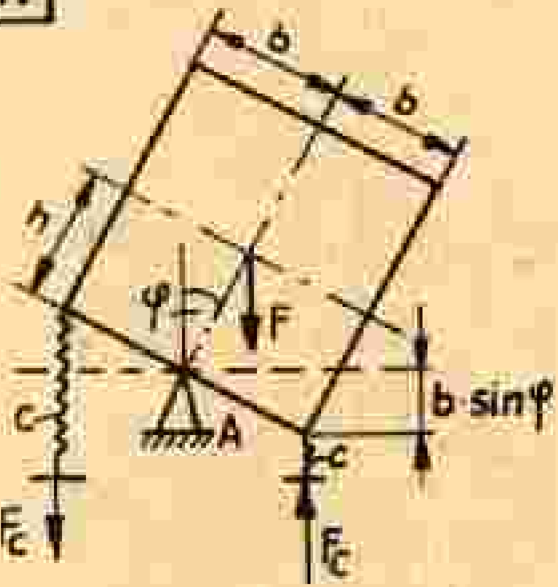
9.1



- Verwendung der Theorie 2. Ordnung (kleine Verformung, Gleichgewichtsbedingungen am verformten Teil)

A: $F \cdot h \sin\varphi - 2c \cdot b \sin\varphi \cdot b \cos\varphi = 0$ $\qquad F_c \cdot b \sin\varphi$

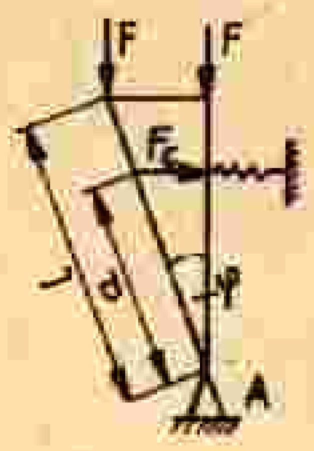
$$F \cdot h \sin\varphi - 2cb^2 \sin\varphi \cos\varphi = 0$$

$$F = \frac{2cb^2 \cancel{\sin\varphi} \underbrace{\cos\varphi}_{\approx 1}}{h \cancel{\sin\varphi}}$$

$$\underline{\underline{F_{kr} = \frac{2cb^2}{h}}}$$

$\boxed{9.2}$

ausgelenkte Lage betrachten



$$\overset{\frown}{A}: -Fl\sin\varphi + a\sin\varphi\, ca\cos\varphi = 0$$

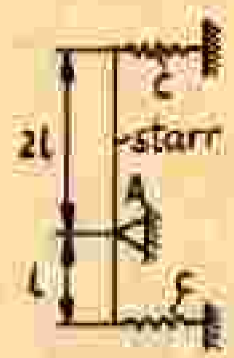
$$F_c = c\sin\varphi \cdot a$$

$$F_{kr} = \frac{a\,c\cancel{\sin\varphi}\cdot\cos\varphi\cdot a}{l\cancel{\sin\varphi}}$$

$$\underline{F_{kr} = \frac{a^2c}{l}}$$

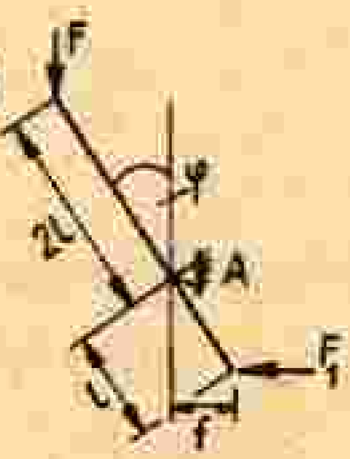
$\boxed{9.3}$

$$\underline{f = \frac{Fl^3}{3EI} = \frac{F}{c}}$$



biegsam eingespannter Träger

a)



$$\overset{\frown}{A}: -F\sin\varphi\cdot 2l + F_1\cos\varphi\cdot l = 0$$

$$-F\cancel{\sin\varphi}\cdot 2l + \frac{6\cdot 3EI\,\cancel{\sin\varphi}\,l\cos\varphi\cdot l}{l^3} = 0$$

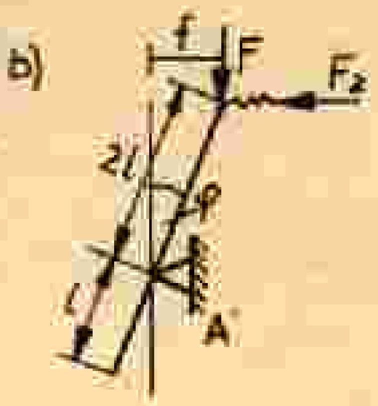
$$-2Fl + \frac{18EI}{l}\cos\varphi = 0$$

$$F = \frac{18EI\cancel{\cos\varphi}}{2l^2}$$

$$\underline{F = \frac{9EI}{l^2}}$$

$\boxed{9.3}$

b)



$$\overset{\frown}{A}: F\,2l\sin\varphi - 2lF_2\cos\varphi = 0$$

$$2lF\cancel{\sin\varphi} - \frac{3EI}{l^3}\cancel{\sin\varphi}\,2l\cos\varphi\cdot 2l = 0$$

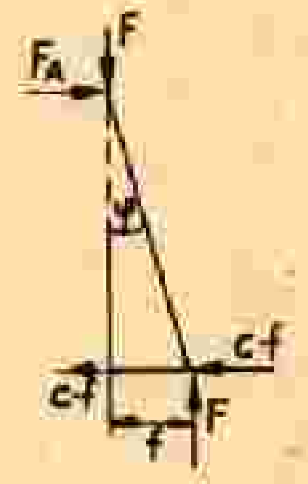
$$2lF - \frac{3EI\cdot 4\cos\varphi}{l} = 0$$

$$F_2 = \frac{12EI\cos\varphi}{2l^2}$$

$$\underline{\underline{F_2 = \frac{6EI}{l^2} = F_{kr}}}$$

$\underline{\underline{F_{kr} = \frac{6EI}{l^2}}}$ $\Longrightarrow$ minimale kritische Last

$\boxed{9.4}$



$$\overset{\frown}{A}: F\sin\varphi\cdot l - 2c\cdot l\sin\varphi\cos\varphi\cdot l = 0$$

$$F = \frac{2cl^{\cancel{2}}\cancel{\sin\varphi}\cancel{\cos\varphi}}{\cancel{l}\cancel{\sin\varphi}}$$

$$\underline{\underline{F = 2cl}}$$

$\boxed{9.5}$

4. Euler-Fall

$l_K = \frac{1}{2}l$

$$F = \frac{\pi^2 EI}{l_K^2 S_K} = \frac{4\pi^2 EI}{l^2 S_K}$$

2. Euler-Fall

$l_K = l$

$$F = \frac{\pi^2 EI}{l_K^2 S_K} = \frac{\pi^2 EI}{l^2 S_K}$$

$\boxed{9.5}$

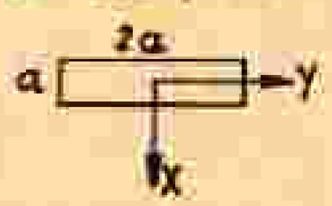
Kreisquerschnitt $l_K = l$

$$I_K = \frac{\pi}{64} d^4 = \frac{F \, S_K l_K^2}{\pi^2 E}$$

$$d = \sqrt[4]{\frac{64 F \, S_K l_K^2}{\pi^3 E}} = \sqrt[4]{\frac{64 \cdot 1000 \cdot 5 \cdot 10^6}{\pi^3 \, 2{,}0 \cdot 10^5}}$$

d = 15,07 mm, gewählt d = 16 mm

Rechteckquerschnitt



$$I_x = \frac{a(2a)^3}{12} \longrightarrow I_x = \frac{2}{3}a^4;$$

$$I_y = \frac{a^3 2a}{12} \longrightarrow I_y = \frac{1}{6}a^4$$

$l_K = \frac{l}{2}$: $I_{y\,erf} = \frac{F S_K \, l^2}{4\pi^2 E} = \frac{1}{6}a^4$

$$a = \sqrt[4]{\frac{3}{2}\frac{F \, S_K l^2}{\pi^2 E}}$$

$l_K = l$: $I_{x\,erf} = \frac{F S_K \, l^2}{\pi^2 E} = \frac{2}{3}a^4$

$$a = \sqrt[4]{\frac{3}{2}\frac{F \, S_K l^2}{\pi^2 E}}$$

In beiden Lagerungsfällen ergibt sich für a derselbe Wert!

$$a = \sqrt[4]{\frac{3 \cdot 1000 \cdot 5 \cdot 10^6}{2\pi^2 \cdot 2 \cdot 10^5}}$$

a = 7,85 mm, gewählt a = 8 mm

$\boxed{9.6}$

Das vorliegende Problem läßt mehrere Lösungswege zu. Hier Lösung - Minimum des elast. Potentials -.

Arbeit der Kraft F: $W_F = F \cdot p$

Energie der Federn: $W_C = \frac{1}{2}c\,(v_1^2 + v_2^2 + v_3^2)$.

Weg der Kraft: $p = p_1 + p_2 + p_3 + p_4$.

mit $p_1 = l - \sqrt{l^2 - v_1^2} \approx l - l\left(1 - \frac{1}{2}\left(\frac{v_1}{l}\right)^2\right) = \frac{1}{2}\frac{v_1^2}{l}$

analog $p_2 = \frac{1}{2}\frac{(v_2 - v_1)^2}{l}$,

$$p_3 = \frac{1}{2}\frac{(v_3 - v_2)^2}{l}$$

$$p_4 = \frac{1}{2}\frac{v_3^2}{l}$$

Elast. Potential $\Pi = W_C - W_F$:

$$\Pi = \frac{1}{2}c(v_1^2 + v_2^2 + v_3^2) - \frac{F}{2l}\left[v_1^2 + (v_2 - v_1)^2 + (v_3 - v_2)^2 + v_3^2\right]$$

Forderung: Π = Min!

$$\frac{\partial \Pi}{\partial v_1} = c\,v_1 - \frac{F}{l}(2v_1 - v_2) = 0\,,$$

$$\frac{\partial \Pi}{\partial v_2} = c\,v_2 - \frac{F}{l}(-v_1 + 2v_2 - v_3) = 0\,,$$

$$\frac{\partial \Pi}{\partial v_3} = c\,v_3 - \frac{F}{l}(-v_2 + 2v_3) = 0\,,$$

9.6

Matrizeneigenwertgleichung

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \frac{cl}{F} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} ; \quad \frac{cl}{F} = \lambda$$

Die Lösung von Eigenwertproblemen mit einer Matrix der Form

$$A = \begin{bmatrix} a & -1 & 0 & 0 & \vdots \\ -1 & a & -1 & 0 & \vdots \\ 0 & -1 & a & -1 & \vdots \\ 0 & 0 & -1 & a & \vdots \\ \cdots & \cdots & \cdots & \cdots & \end{bmatrix} ; \quad \text{(Matrix A vom Format } n \times n\text{)}$$

lautet (siehe z.B. Zurmühl, „Matrizen", S. 229)

mit $\varphi_k = \frac{k\pi}{n+1}$

$\lambda_k = a - 2\cos\varphi_k ,$

$v_{ki} = \sin\frac{ki\pi}{n+1} ,$

hier mit $n = 3$, $a = 2$

$$\lambda_k = 2\left(1 - \cos\frac{k\cdot\pi}{4}\right),$$

$$v_{ki} = C_k \sin\left(\frac{ik\pi}{4}\right)$$

Infrage kommt hier der größte Eigenwert

$\lambda_1 = 2\left(1 - \frac{\sqrt{2}}{2}\right)$

$\lambda_2 = 2$

$\lambda_3 = 2\left(1 + \frac{\sqrt{2}}{2}\right)$, also $\lambda_{krit} = \lambda_3$

9.6

Es folgt:

$$\underline{F_{kr} = \frac{cl}{2+\sqrt{2}}}$$

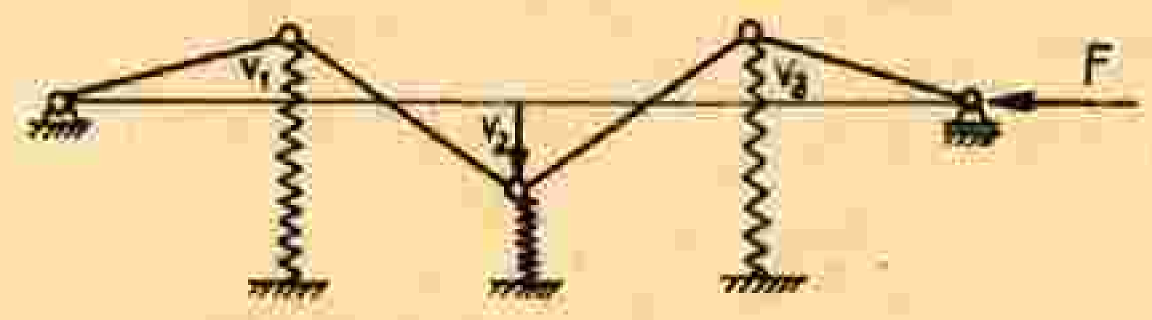
Die zugehörige Eigenfunktion lautet ($k = 3$, $C_3 = 1$)

$v_1 = \sin\left(\frac{3\pi}{4}\right) = +\frac{\sqrt{2}}{2}$

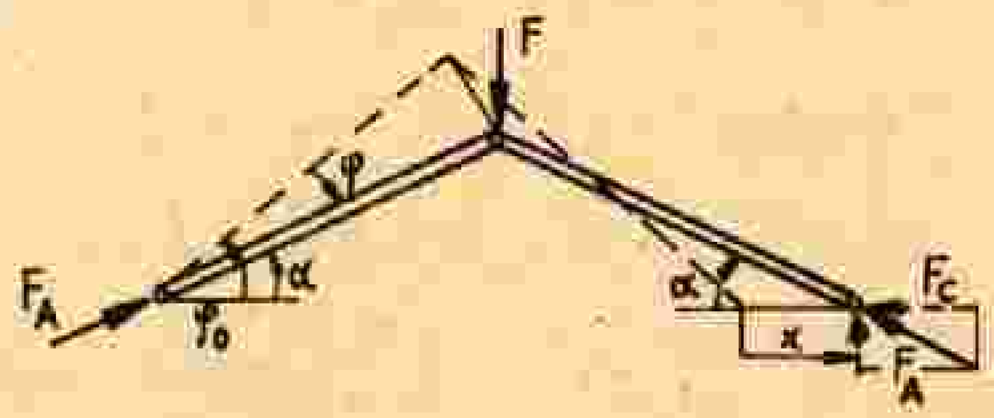
$v_2 = \sin\left(\frac{6\pi}{4}\right) = -1$

$v_3 = \sin\left(\frac{9\pi}{4}\right) = +\frac{\sqrt{2}}{2}$

Knickfigur



9.7



1. $\alpha = \varphi_0 - \varphi$

$\uparrow: -F + 2F_A \sin\alpha = 0 \Rightarrow F_A = \frac{F}{2\sin(\varphi_0 - \varphi)}$,

$F_C = F_A \cdot \cos\alpha \Rightarrow F_C = \frac{F}{2\tan(\varphi_0 - \varphi)}$,

9.7

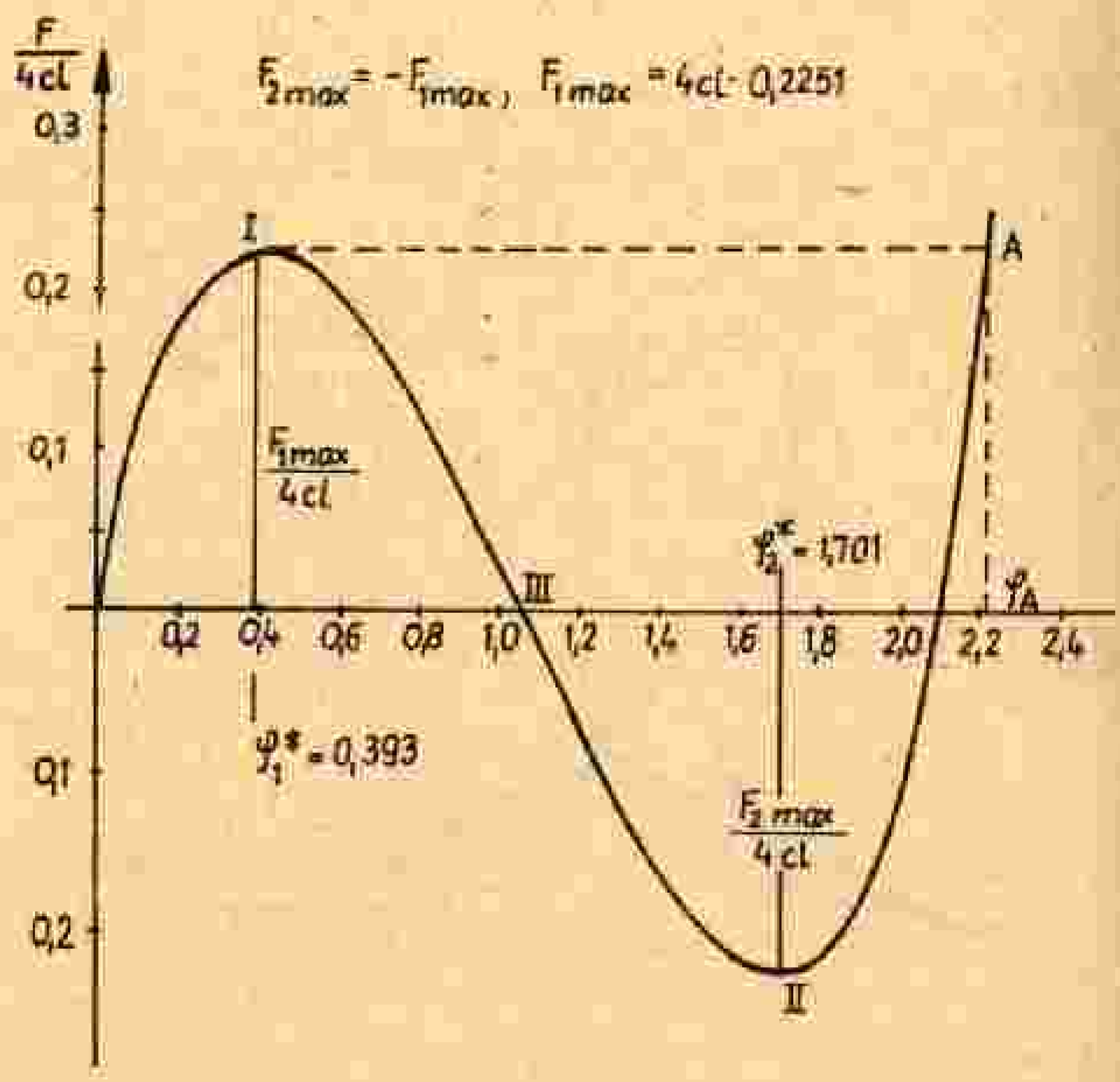
Federweg x: $x = 2l\left[\cos(\varphi_0-\varphi)-\cos\varphi_0\right]$

$$F_c = c\cdot x$$

$$\frac{F}{2\tan(\varphi_0-\varphi)} = 2cl\left[\cos(\varphi_0-\varphi)-\cos\varphi_0\right]$$

$$F = 4cl\left[\cos(\varphi_0-\varphi)-\cos\varphi_0\right]\cdot\tan(\varphi_0-\varphi)$$

2) $\varphi_0 = \frac{\pi}{3}$



9.7

3) Bei φ_1^* erfolgt Durchschlagen der Last F, φ vergrößert sich auf den Wert φ_A und das System erreicht wieder eine stabile Gleichgewichtslage.
Wird der Lastpunkt jedoch zwangsweise geführt, so stellt sich zu jedem φ die Last entsprechend der graphischen Darstellung ein.

4) $$\frac{dF}{d\varphi} = 0 = 4cl\left\{\sin(\varphi_0-\varphi)\cdot\tan(\varphi_0-\varphi)+\left[\cos(\varphi_0-\varphi)-\cos\varphi_0\right]\cdot\frac{-1}{\cos^2(\varphi_0-\varphi)}\right\}$$

vereinfacht:

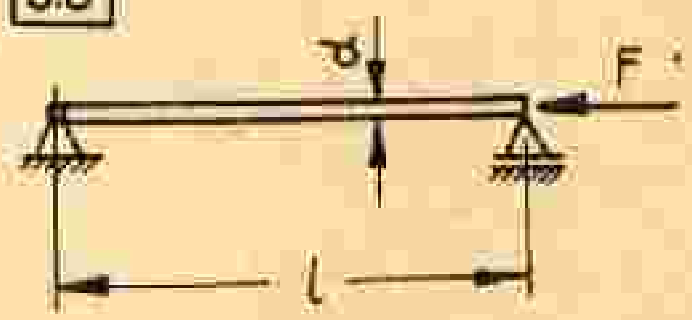
$$\cos\varphi_0 - \cos^3(\varphi_0-\varphi) = 0$$

$$\varphi = \varphi_0 - \arccos\left(\sqrt[3]{\cos\varphi_0}\right)$$

$$\arccos\left(\sqrt[3]{0{,}5}\right) = \pm 0{,}653928$$

$$\varphi_1^* = 0{,}39327,\quad \varphi_2^* = 1{,}70113$$

9.8



$\sigma = \frac{F}{A}$; $A = \frac{\pi}{4}d^2$

$\sigma_k = \frac{F}{A}\cdot S_k$

Schlankheitsgrad $\lambda = \frac{l_k}{i}$, 2. Eulerfall $l_k = l$

Kreisquerschnitt: $i = \frac{d}{4}$

$$\lambda = \frac{4l}{d}$$

$\boxed{9.8}$

elast. Knicken: $d = d_e$

$$\sigma_k = \frac{\pi^2 E}{\lambda^2} \Rightarrow \frac{4F\, S_K}{\pi\, d_e^2} = \frac{\pi^2 E}{16 l^2}\, d_e^2$$

$$d_e = \sqrt[4]{\frac{64\, F S_K\, l^2}{\pi^3 E}} = \sqrt[4]{\frac{64\cdot 20\cdot 10^3\cdot 5\cdot 500^2}{\pi^3\cdot 2{,}0\cdot 10^5}}$$

$$\underline{d_e = 22{,}54\ \text{mm}}$$

elast.-plast. Knicken: $d = d_p$

$$\sigma_k = 310 - 1{,}14\cdot\lambda \Rightarrow \frac{4F\cdot S_K}{\pi\, d_p^2} = 310 - 1{,}14\cdot\frac{4l}{d_p}$$

$$d_p^2\cdot 310 - 2280\cdot d_p - 1{,}2732\cdot 10^5 = 0$$

$$d_p^2 - 7{,}355\, d_p - 410{,}71 = 0$$

$$\underline{d_p = 24{,}28\,\text{mm}} \Rightarrow \underline{d = 25\ \text{mm gewählt}}$$

$$\lambda = \frac{4l}{d} = 80$$

Grenzschlankheitsgrad:

$$310 - 1{,}14\lambda = \frac{\pi^2 E}{\lambda^2}$$

Gleichung 3. Grades für $\lambda = \lambda_p$

Lösung: $\underline{\lambda_p = 100{,}5}$

$\boxed{9.9}$

$$\underline{F_k = \frac{\pi^2 E\cdot I}{l_k^2}} \qquad \underline{I_{\text{▭}}} = \frac{2b\cdot b^3}{12} = \underline{\frac{b^4}{6}} \qquad \boxed{l_k = l}$$

$$F_{k\,\text{▭}} = \frac{\pi^2 E\, b^4}{6 l^2} \qquad \underline{I_{\square} = \frac{a^4}{12}}$$

$$F_{k\,\square} = \frac{\pi^2 E\, a^4}{12 l^2} \qquad \underline{I_{\circ} = \frac{\pi}{64} d^4}$$

$$F_{k\,\circ} = \frac{\pi^3 E\, d^4}{64 l^2} \qquad \underline{I_{\odot} = \frac{\pi}{64}\left(da^4 - (0{,}7\,da)^4\right)}$$

$$F_{k\,\odot} = \frac{\pi^2 E\left(da^4 - (0{,}7\,da)^4\right)}{64 l^2}$$

$$A_{\text{▭}} = 2b^2 \qquad A_{\square} = a^2$$

$$2b^2 = a^2$$

$$b^2 = \frac{a^2}{2} \qquad \underline{b^4 = \frac{a^4}{4}}$$

$$\underline{\frac{F_{k\,\square}}{F_{k\,\text{▭}}} = 2}$$

$$\underline{\frac{F_{k\,\circ}}{F_{k\,\text{▭}}} = \frac{6}{\pi}}$$

$$A_{\text{▭}} = 2b^2$$

$$A_{\circ} = \frac{\pi}{4} d^2$$

$$2b^2 = \frac{\pi}{4} d^2$$

$$\underline{b^2 = \frac{\pi}{8} d^2}$$

$$\underline{\frac{F_{k\,\odot}}{F_{k\,\text{▭}}}} = \frac{0{,}7599}{0{,}2601}\cdot\frac{6}{\pi} = \underline{5{,}579}$$

$$A_{\odot} = \frac{\pi}{4}\left(da^2 - (0{,}7\,da)^2\right) = \frac{\pi}{4}\cdot 0{,}51\cdot da^2$$

$$A_{\text{▭}} = 2b^2$$

$$\underline{b^2 = \frac{\pi}{8}\cdot 0{,}51\, da^2}$$

$\boxed{9.10}$

Schlankheitsgrad

$\underline{\lambda = \frac{l}{i}}$ $\quad \underline{i} = \sqrt{\frac{I_{vorh}}{A_{vorh}}} = \sqrt{\frac{2700\ mm^4}{300\ mm^2}} = \underline{3mm}$

$\underline{\lambda_1 = \lambda_{min}} = \frac{700mm}{3\ mm} = \underline{233{,}33}$ $\qquad \underline{l_K = l}$

$\sigma_F - \lambda_F$ Fließgrenze / Streckgrenze

$\sigma_P - \lambda_P$ Proportionalitätsgrenze

$\sigma_K = \alpha - \beta\lambda$ St 38 $\quad \lambda_P = 105 \quad \alpha = 3100$

$\beta = 11{,}4$

$\alpha_{th} = 12{,}2 \cdot 10^{-6}\ K^{-1}$

$\sigma_{d(K)} = \sigma_K = \frac{F_K}{A} = \pi^2 \frac{EI}{s_K^2 A} \qquad \lambda = \frac{s_K}{i} \qquad s_K = l$

$\sigma_K = \frac{F_K}{A} = \pi^2 \frac{EI}{l^2 A} \qquad \lambda = \frac{l}{i} = \frac{\pi}{\pi}\sqrt{\frac{Al^2}{I}}$

$\underline{\sigma_K = E\left(\frac{\pi}{\lambda}\right)^2} \qquad \underline{\sigma_{d(K)} = \sigma_F}$

Druckspannung

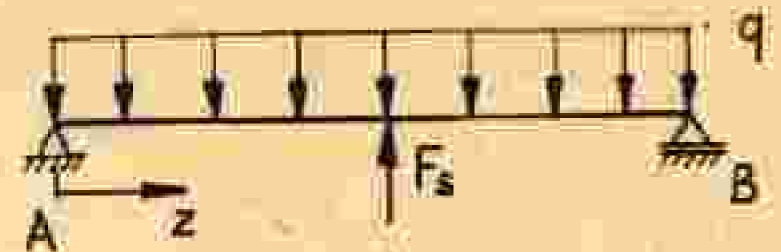
$\rightarrow \sigma_F = E\frac{\pi^2}{\lambda^2}$

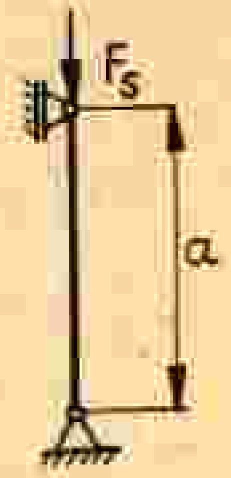
b) $\underline{l_K = \frac{1}{2}\sqrt{2}\,l}$

$\underline{\lambda_2} = \frac{1}{2}\sqrt{2} \cdot \frac{700\,mm}{3\ mm} = \underline{164{,}99}$

c) $\underline{l_K = \frac{1}{2} l} \rightarrow \underline{\lambda_3} = \frac{1}{2} \cdot \frac{700}{3} = \underline{116{,}66}$

$\boxed{9.11}$





Träger auf 3 Stützen:

$F_S = \frac{5}{4} q \cdot a$

2 Eulerfall $\quad l_K = a$

$F_K = \frac{\pi^2 \cdot EI_2}{a^2} = \frac{5}{4} q_K \cdot a$

$q_K = \frac{4}{5} \frac{\pi^2 EI_2}{a^3} = \frac{4}{5} \frac{\pi^2 \cdot 2\ 10^5 \cdot 1{,}17 \cdot 10^6}{2{,}5^3 \cdot 10^9}$

$\underline{q_K = 118{,}2\ N/mm}$

Für $q > q_K$ bleibt $F = F_K \approx$ konst.

$F_{AV} = q \cdot a - \frac{5}{8} q_K \cdot a$

$\underline{M_b = \left(q\,a - \frac{5}{8} q_K a\right) z - \frac{1}{2} q z^2}$

$\underline{M_b(z = a) = \frac{1}{2} q a^2 - \frac{5}{8} q_K a^2}$

$\boxed{9.12}$

$E = 2{,}0 \cdot 10^5\ N/mm^2$

$\sigma_p = \frac{\pi^2 E}{\lambda_p^2} = a - b\lambda_p$

Gleichung 3. Grades, Lösung: $\lambda_p = 98{,}2$

2. Eulerfall

$$i = \sqrt{\frac{I}{A}} \qquad I = \frac{\pi}{64} d^4 \qquad A = \frac{\pi}{4} d^2$$

$$\underline{i} = \sqrt{\frac{\pi \cdot d^4 \cdot 4}{64 \pi \cdot d^2}} = \sqrt{\frac{d^2}{16}} = \frac{d}{4} = \frac{30}{4} = \underline{7{,}5\ mm}$$

$$\underline{\lambda} = \frac{l_k}{i} = \frac{480}{7{,}5} = \underline{64} \qquad l_k = l$$

$\lambda < \lambda_p \Rightarrow$ plastisches Verhalten

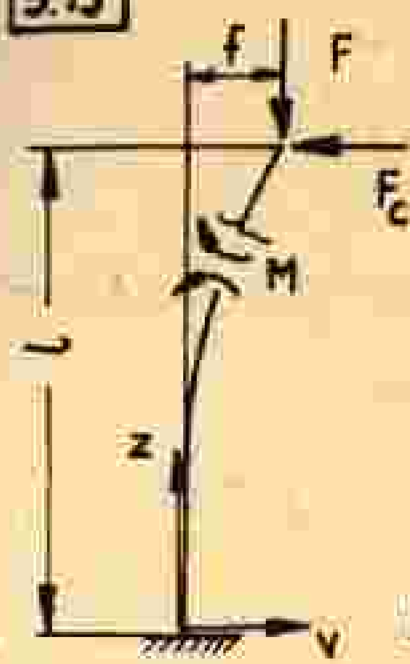
Verwenden der Tetmajer-Gerade

$$\underline{\sigma_{kr}} = a - b\lambda = 283{,}5 - 0{,}802 \cdot 64 = \underline{232{,}172\ N/mm^2}$$

$$\underline{\sigma_{Dvorh}} = p\,\frac{\frac{\pi \cdot D^2}{4}}{\frac{\pi d^2}{4}} = p\,\frac{D^2}{d^2} = 7 \cdot 10^5 \cdot \frac{4000}{900} = \underline{31{,}11\ N/mm^2}$$

$$\underline{S_K} = \frac{\sigma_{kr}}{\sigma_{Dvorh}} = \frac{232{,}172\ N/mm^2}{31{,}11\ N/mm^2} = \underline{7{,}462}$$

$\boxed{9.13}$



Gleichgewicht

$$M = -F(f-v) + F_c(l-z)$$

Biegelinie

$$M = -EI\,v''$$

Dgl. des ausgebogenen Stabes

$$EI\,v'' + F \cdot v = F f - F_c l + F_c z$$

mit $\alpha^2 = \frac{F}{EI}$ folgt

$$v'' + \alpha^2 v = \alpha^2 f - \alpha^2 \frac{F_c}{F} l + \alpha^2 \frac{F_c}{F} \cdot z$$

Lösung:

$$v = A \cos\alpha z + B \sin\alpha z + f - \frac{F_c}{F} l + \frac{F_c}{F} \cdot z$$

$$F_c = f \cdot c$$

Randbedingungen

1) $v(0) = 0 \Rightarrow A + f - \frac{c f}{F} l = 0$

2) $v'(0) = 0 \Rightarrow \alpha B + \frac{c f}{F} = 0$

3) $v(l) = f \Rightarrow A \cos\alpha l + B \sin\alpha l = 0$

Es folgt die Eigenwertgleichung

$$\left[-(\alpha l)^2 \frac{EI}{c l^3} + 1\right] \cos\alpha l - \frac{1}{\alpha l} \sin\alpha l = 0$$

für $\frac{EI}{c l^3} = 0{,}01\ ;\ 0{,}1\ ;\ 1$ folgt

$$\alpha l = 4{,}44;\ 3{,}16;\ 1{,}81$$

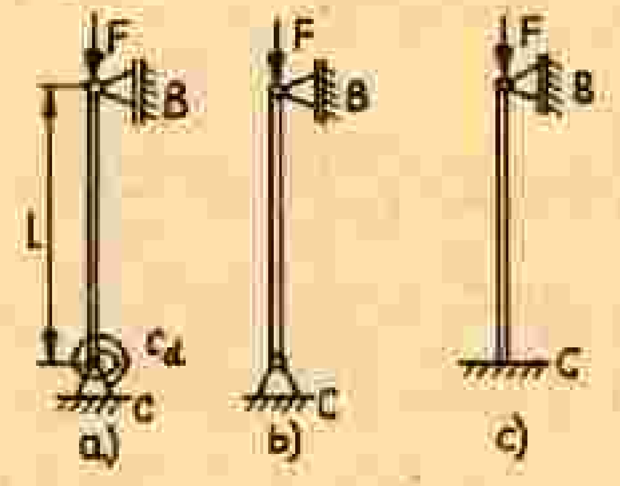
$\boxed{9.13}$

Für $c \Rightarrow 0$ folgt $\cos\alpha l = 0$ (1. Eulerfall),

$c \Rightarrow \infty$ folgt $\cos\alpha l - \frac{1}{\alpha l}\sin\alpha l = 0$ (3. Eulerfall).

$\boxed{9.14}$

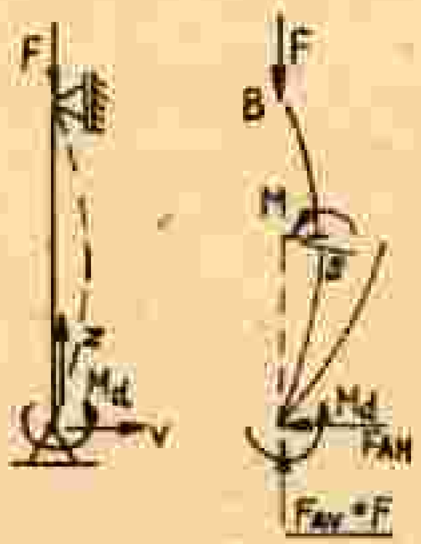
Knickstab mit verschiedenen Lagerungsarten



geg: l, EI, C_d

Lösung

zu 1



$M_d = C_d\, v_0' = C_d\, \varphi_0$

(B): $F_{AH} = \frac{M_d}{l}$

(S): $-M + F_v - M_d + F_{AH}\, z = 0$

$EIv'' + Fv = -M_d - F_{AH}\, z$

$EIv'' + Fv = C_d\, \varphi_0 \left[1 - \frac{z}{l}\right]$

$\varkappa^2 = \frac{Fl^2}{EI}$

$\boxed{9.14}$

Lösung: $v = A\cos\varkappa\frac{z}{l} + B\sin\varkappa\frac{z}{l} + \frac{C_d\varphi_0}{F}\left[1-\frac{z}{l}\right]$

RB ① $v(0) = 0 \rightarrow A + \frac{C_d\varphi_0}{F} = 0$

$v(l) = 0 \rightarrow A\cos\varkappa + B\sin\varkappa = 0$

$v'(0) = \varphi_0 \rightarrow \frac{\varkappa}{l}B - \left[\frac{C_d}{Fl}+1\right]\varphi_0 = 0$

Koeffizientendeterminante = 0

$$\begin{vmatrix} 1 & 0 & \frac{C_d}{F} \\ \cos\varkappa & \sin\varkappa & 0 \\ 0 & \frac{\varkappa}{l} & -\left[\frac{C_d}{Fl}+1\right] \end{vmatrix} = 0$$

$$-\frac{C_d}{Fl}\sin\varkappa - \sin\varkappa + \frac{\varkappa\, C_d}{l\, F}\cos\varkappa = 0$$

mit $F = \frac{\varkappa^2 EI}{l^2}$

$$-\frac{C_d l^2}{\varkappa^2 l EI}\sin\varkappa - \sin\varkappa + \frac{\varkappa C_d l^2}{\varkappa^2 l EI}\cos\varkappa = 0$$

$$-1 - \frac{\varkappa^2 EI}{C_d l} + \varkappa\cot\varkappa = 0 \qquad K = \frac{EI}{C_d l}$$

$$-\frac{1}{\varkappa} - \varkappa k + \cot\varkappa = 0$$

mit $\varkappa = \alpha l$

$$-\frac{1}{\alpha l} - \alpha l K + \cot(\alpha l) = 0$$ Eigenwertgl.

k	αl
0,01	4,449
0,1	4,1323
1	3,4056

$\boxed{9.14}$

<u>Fall b</u>: Eigenwertgleichung umgeformt

$$\left[-\frac{\sin\alpha l}{\alpha l}+\cos\alpha l\right]\frac{c_d l}{EI}-\alpha l\sin\alpha l=0$$

für $c_d = 0$ folgt

$$\underline{\sin\alpha l = 0} \quad \text{(Eigenwertgleichung 2. Eulerfall)}$$

Fall c

für $c_d \Rightarrow \infty$ folgt

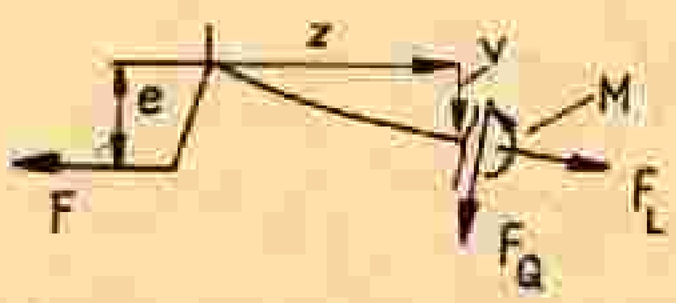
$$\underline{\alpha l = \tan\alpha l} \quad \text{(Eigenwertgleichung 3. Eulerfall)}$$

$\boxed{9.15}$

Anwendung der Theorie 2. Ordnung

$$M_b = F(e-v)$$

$$v'' = -\frac{M}{EI}$$



$$v'' = -\frac{F}{EI}(e-v)$$

Mit $\alpha^2 = \frac{F}{EI}$ ergibt sich die Differentialgleichung

$$v'' - \alpha^2 v = -\alpha^2 e$$

$\boxed{9.15}$

Lösung

$$v = C_1\cosh(\alpha z) + C_2\sinh(\alpha z) + e \qquad \text{für } F > 0$$

$$v = C_1\cos(\bar\alpha z) + C_2\sin(\bar\alpha z) + e \qquad \text{für } F < 0$$

$$(\bar\alpha = -\alpha)$$

Randbedingungen

1) $v(0) = 0$, 2. $v'(z=\frac{l}{2}) = 0$ (Symmetrie)

<u>$F > 0$</u>

$$\left.\begin{aligned} &C_1 + e = 0 \\ &C_1\sinh\frac{\alpha l}{2} + C_2\cosh\frac{\alpha l}{2} = 0 \end{aligned}\right\} \begin{aligned} &C_1 = -e \\ &C_2 = e\,\frac{\sinh\frac{\alpha l}{2}}{\cosh\frac{\alpha l}{2}} \end{aligned}$$

<u>$F < 0$</u>

$$\left.\begin{aligned} &C_1 + e = 0 \\ &-C_1\sin\frac{\bar\alpha l}{2} + C_2\cos\frac{\bar\alpha l}{2} = 0 \end{aligned}\right\} \begin{aligned} &C_1 = -e \\ &C_2 = -e\,\frac{\sin\frac{\bar\alpha l}{2}}{\cos} \end{aligned}$$

Max. Biegemoment und max. Zugspannung

<u>$F > 0$</u>

$$M_b(z=0) = F\cdot e = M_{b\,max}$$

$$M_b(z=\tfrac{l}{2}) = F\frac{e}{\cosh\frac{\alpha l}{2}} < F\cdot e \quad ; \quad e = \frac{h}{2}$$

$$\sigma_{max} = \frac{F}{A} + \frac{Fe}{I}\cdot\frac{h}{2} = \underline{\underline{\frac{F}{A}\left(1+\frac{Ah^2}{4I}\right)}}$$

$\boxed{9.15}$

$\underline{F < 0}$

$M_b(z=0) = F\,e$

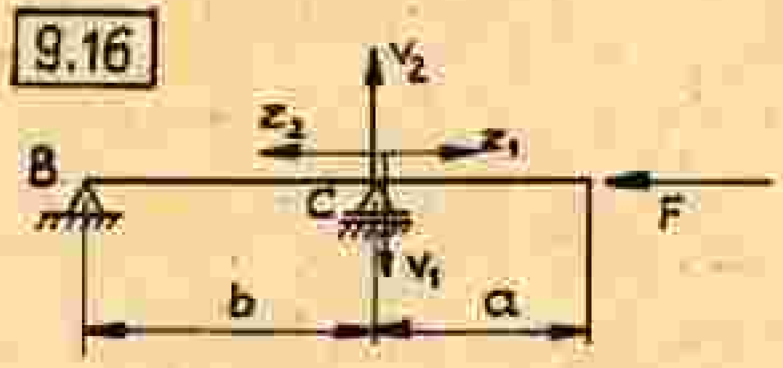
$M_b\left(z=\frac{l}{2}\right) = F\frac{e}{\cos\left(\frac{\alpha l}{2}\right)}$; $\left|M_b\left(z=\frac{l}{2}\right)\right| > F_e$

$\alpha l \Rightarrow \pi$ ergibt $|M_b| \Rightarrow \infty$ (F = Knicklast)

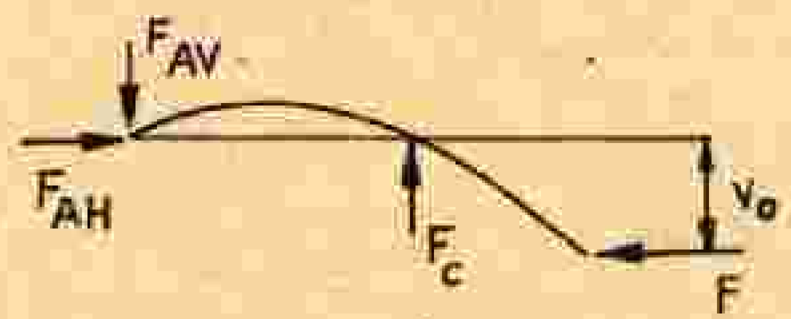
$\sigma_{max} = \frac{F}{A} - \frac{Fe}{I}\cdot\frac{h}{2}\,\frac{1}{\cos\frac{\alpha l}{2}}$ $\quad (F<0)$

$$\sigma_{max} = \frac{|F|}{A}\left(\frac{Ah^2}{4I}\,\frac{1}{\cos\left(\frac{\alpha l}{2}\right)} - 1\right)$$

$\boxed{9.16}$



Anwendung der Theorie 2. Ordnung



$F_{AH} = F$

$F_{AV} = F\,\frac{v_0}{b} = F_c$

$\boxed{9.16}$

Biegemomente :

$M_{b1} = -F(v_0 - v_1)$ (1. Abschnitt)

$M_{b2} = +F v_2 + F\frac{v_0}{b}(b - z_2)$ (2. Abschnitt)

$v_1'' = \frac{F}{EI}(v_0 - v_1)$; $v_2'' = -\frac{F}{EI}\left(v_2 + v_0 - v_0\,\frac{z_2}{b}\right)$

Lösung der Differentialgleichungen: $\underline{\alpha^2 = \frac{F}{EI}}$

$v_1 = A_1\cos(\alpha_1 z_1) + B_1\sin(\alpha_1 z_1) + v_0$

$v_2 = A_2\cos(\alpha z_2) + B_2\sin(\alpha z_2) + \frac{v_0}{b}\left(\frac{z_2}{b} - 1\right)$

Rand - und Übergangsbedingungen

1.) $v_1(0) = 0 \Rightarrow A_1 = -v_0$

2.) $v_2(0) = 0 \Rightarrow A_2 = v_0$

3.) $v_2(b) = 0 \Rightarrow v_0\cos(\alpha b) + B_2\sin(\alpha b) = 0$

4.) $v_1(a) = v_0 \Rightarrow -v_0\cos(\alpha a) + B_1\sin(\alpha a) = 0$

5.) $v_1'(0) = v_2'(0) \Rightarrow \alpha B_2 + \frac{v_0}{b} = \alpha B_1$

Die Koeffizientendeterminante der B_1, B_2, v_0 muß verschwinden

$$\begin{vmatrix} 0 & \sin(\alpha b) & \cos(\alpha b) \\ \sin(\alpha a) & 0 & -\cos(\alpha a) \\ \alpha b & -\alpha b & -1 \end{vmatrix} = 0$$

9.16

Entwicklung nach den Elementen der letzten Zeile

$$-\alpha a \sin(\alpha b)\cos(\alpha a) - \alpha b \cos(\alpha b)\sin(\alpha a) + \sin(\alpha a)\sin(\alpha b) = 0$$

$$\underline{\underline{\alpha b \cot(\alpha a) + \alpha b \cot(\alpha b) = 1}}$$

Für $a = b$ folgt

$$2\alpha b \cot(\alpha b) = 1$$

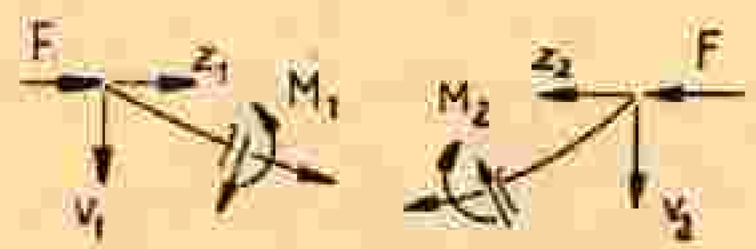
bzw $\underline{2\alpha b = \tan(\alpha b)}$

Lösung $\alpha b = 1{,}1656$

$$F_k = (\alpha b)^2 \frac{EI}{b^2} = \underline{\underline{0{,}1376 \frac{\pi^2 EI}{b^2}}}$$

9.17

Geg.: E, I_1, I_2, l, l_1



$M_1 = F v_1$

$M_2 = F v_2$

$$\alpha_1^2 = \frac{F}{EI_1} \qquad \alpha_2^2 = \frac{F}{EI_2}$$

9.17

DGL $v_1'' + \alpha_1^2 v_1 = 0 \qquad v_2'' + \alpha_2^2 v_2 = 0$

Lösung: $v_1 = C_1 \cos(\alpha_1 z_1) + C_2 \sin(\alpha_1 z_1)$

$$v_2 = D_1 \cos(\alpha_2 z_2) + D_2 \sin(\alpha_2 z_2)$$

RB: ① $v_1(0) = 0 \longrightarrow C_1 = 0$

② $v_2(0) = 0 \longrightarrow D_1 = 0$

③ $v_1(l_1) = v_2(l - l_1) \longrightarrow C_2 \sin(\alpha_1 l_1) - D_2 \sin[\alpha_2(l-l_1)] = 0$

④ $v_1'(l_1) = -v_2'(l-l_1) \longrightarrow \alpha_1 C_2 \cos(\alpha_1 l_1) + \alpha_2 D_2 \cos[\alpha_2(l-l_1)] = 0$

Koeffizientendeterminante = 0

$$\begin{vmatrix} \sin(\alpha_1 l_1) & -\sin[\alpha_2(l-l_1)] \\ \alpha_1 \cos(\alpha_1 l_1) & \alpha_2 \cos[\alpha_2(l-l_1)] \end{vmatrix} = 0$$

aus Koeffizientendeterminante = 0

$$\Rightarrow$$

$$\sin(\alpha_1 l_1)\,\alpha_2 \cos[\alpha_2(l-l_1)] + \alpha_1 \cos(\alpha_1 l_1) \sin[\alpha_2(l-l_1)] = 0$$

$$\frac{\alpha_2 \sin(\alpha_1 l_1) \cos[\alpha_2(l-l_1)]}{\alpha_1 \cos(\alpha_1 l_1)} + \sin[\alpha_2(l-l_1)] = 0$$

$$\underline{\underline{\frac{\alpha_2}{\alpha_1} \tan(\alpha_1 l_1) + \tan[\alpha_2(l-l_1)] = 0}} \quad \text{Eigenwertgleichung}$$

<u>zu 2.</u> $F_k = \alpha_1^2 EI_1 \qquad \alpha_2^2 = \frac{F_k}{EI_2}$

$$\Rightarrow$$

$$\alpha_2^2 = \alpha_1^2 \frac{I_1}{I_2}$$

$$\underline{\alpha_2 = \alpha_1 \sqrt{\frac{I_1}{I_2}}}$$

mit $I_1 = 0{,}49\, I_2$

$$\Rightarrow$$

$$\sqrt{0{,}49}\tan(\alpha_1 l_1) + \tan[\alpha_1 \sqrt{0{,}49}\,(l - l_1)] = 0$$

mit $l_1 = 0{,}4 l$

$$\underline{l - l_1} = \frac{l_1 - 0{,}4 l_1}{0{,}4} = \underline{1{,}5 l_1}$$

$$\Rightarrow$$

$$\underline{0{,}7 \tan(\alpha_1 l_1) + \tan(1{,}05\,\alpha_1 l_1) = 0}$$

$$\Rightarrow \underline{\underline{F_k = 1{,}5 \frac{\pi^2 EI_1}{l^2}}}$$